An Introduction to Algebraic Physics
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Introduction

The aim of this document is to start using algebraic notions and tools of many-valued reasoning, in the guise of MV-algebras, to investigate the Mathematical Universe Hypothesis (MUH). The hope of the author is in settling the conjectures, a deeper understanding of the universe is realized.

Part I

Key Concepts and some Conjectures

Notation. Let Γ_l be the theory of l-groups with a distinguished order unit, $\mathcal{A}$ be the category of all l-groups with a distinguished order unit, $\mathcal{M}$ be the category of all MV-algebras. As proved in Chapter 7 of [2], there is a natural equivalence between $\mathcal{A}$ and $\mathcal{M}$. Presently, the author uses $\mathbb{F}$ to denote the functor, made explicit in [2] with different notation, that is a natural equivalence between $\mathcal{A}$ and $\mathcal{M}$. Furthermore, $\mathbb{Q}^*$ is the set of hyper-rational numbers, 0 is the integer 0 and the $+$ operation is inherited from the set of hyper-real numbers, as is the usual infinitesimal order on $\mathbb{R}^*$, call it $\leq$.

Conjecture 1. Assuming the MUH, the multiverse studied in Physics is a model of Γ_l.

Conjecture 2. Assuming the MUH, the universe is isomorphic to the following l-group with distinguished order unit 1:

$$G =_{def} \langle \mathbb{Q}^*, +, 0 \rangle .$$

Conjecture 3. Assuming conjecture 2, the logic "most natural" to the universe is the many valued logic with the MV-algebra given by $\mathbb{F}(G, 1)$.

Note that

$$\mathbb{F}(G, 1) =_{def} \left\langle \mathbb{Q}^* \cap [0, 1]^*, \oplus, \neg, 0 \right\rangle ,$$

where the carrier of $\mathbb{F}(G, 1)$, i.e., the truth set of $\mathbb{F}(G, 1)$, is the set of all hyper-rational numbers that are both not less than zero and not greater than one, 0 is the rational number 0, and the operations are defined by the following equations for all $x, y \in \mathbb{Q}^* \cap [0, 1]^*$:

$$x \oplus y =_{def} 1 \wedge (x + y)$$

and

$$\neg x =_{def} 1 - x.$$

Plausibility Case (for conjecture 3). Given that there is a natural equivalence between $\mathcal{A}$ and $\mathcal{M}$ and that in this natural equivalence, the usual functor $\mathbb{F}$ sends G to $\mathbb{F}(G, 1)$.

Remark. Let $\mathcal{T} = \mathbb{Q}^* \cap [0,1]^*$. One could define the set of designated and anti-designated truth degrees in $\mathcal{T}$ by the following equations,

$$D^+ =_{def} \{x \in \mathcal{T} : x \simeq 1\}$$

and

$$D^- =_{def} \{x \in \mathcal{T} : x \simeq 0\},$$

where the symbol $\simeq$ is the usual one for infinitesimal proximity in nonstandard analysis.

Part II
The Universe Generation Operator and Generated Law Operator

Let $\mathcal{R} = \langle R, +, \cdot, 0, 1\rangle$ be a ring with carrier R.

Definition. The universe generated by $\mathcal{R}$, denoted $\mathcal{U}(\mathcal{R})$, is defined to be the group $\langle R, +, 0\rangle$:

$$\mathcal{U}(\mathcal{R}) =_{def} \langle R, +, 0\rangle.$$

Definition. The laws generated by $\mathcal{R}$, denoted $\mathcal{L}(\mathcal{R})$, is defined conditionally:

- If $\mathcal{U}(\mathcal{R})$ is a model for Γ_l, then $\mathcal{L}(\mathcal{R}) =_{def} \mathbb{F}\left(\mathcal{U}(\mathcal{R}), 1_{\mathcal{U}(\mathcal{R})}\right)$ where $1_{\mathcal{U}(\mathcal{R})}$ is a distinguished order unit of $\mathcal{U}(\mathcal{R})$.
- If $\mathcal{U}(\mathcal{R})$ is not a model for Γ_l, then $\mathcal{L}(\mathcal{R}) =_{def} C$ where C is the MV-algebra associated with classical logic.

Conjecture 2 can then be rephrased to be, "assuming the MUH, the universe is isomorphic to $\mathcal{U}(\langle \mathbb{Q}^*, +, \cdot, 0, 1\rangle)$."

Conjecture 3 can then be rephrased to be, "assuming conjecture 2, the logic "most natural" to the universe is the many valued logic with the MV-algebra given by $\mathcal{L}(\langle \mathbb{Q}^*, +, \cdot, 0, 1\rangle)$."

Definition. A worldline with respect to $\mathcal{R}$ is any element of $\mathcal{U}(\mathcal{R})$.

Definition. The pivot operation $\circledcirc$ on $\mathcal{U}(\mathcal{R})$ is defined by the following equation for all worldlines x and y in $\mathcal{U}(\mathcal{R})$:

$$x \circledcirc y =_{def} (x + y) - x \cdot y.$$

$x \circledcirc y$ is read, "x pivot y."

Fix a contextual set theory $\mathfrak{S}$, possibly in the context of any *generalization* of a (not critically relevant to this discussion) classical logic, $\mathfrak{L}$. Then consider the notion of the $\mathfrak{S}$-theory developed in a context in which the axiom schemata of $\mathfrak{S}$ are formulated in the context of $\mathfrak{L}$; notate this $\mathfrak{L}$-dependent $\mathfrak{S}$-theory by the pair $(\mathfrak{L}, \mathfrak{S}) =: \mathfrak{T}$. Whenever some $\mathfrak{T}$-theoretical notions are invoked, they will have a self-explanatory notation such as $\mathsf{card}_{\mathfrak{T}}$. A simple example of such is the $\mathfrak{T}$-theory in which $\mathfrak{L}$ is classical logic and $\mathfrak{S}$ is the $\mathfrak{S}$-theory in which the 'empty set' is the only $\mathfrak{S}$-set.

Part III
Subsequent Investigations and Additional Conjectures

- Investigate the properties of $\mathcal{L}$, which is, by virtue of its definition, a functor between the category of all models and $\mathcal{M}$. *In essence*, $\mathcal{L}$ is a 'go between' from Model Theory and the theory of MV-algebras.

Conjecture 4. The universe is $\mathcal{U}(\mathbb{U})$ where $\mathbb{U}$ is the usual Boolean (lattice) ring associated with an $\mathfrak{T}$-theory with a reasonable notion of a universal set, i.e., an $\mathfrak{S}$-set containing all other $\mathfrak{S}$-sets. In the ring $\mathbb{U}$, the sum operator is disjoint union of $\mathfrak{S}$-sets and multiplication is intersection.

Conjecture 5. The laws of the universe are $\mathcal{L}(\mathbb{U})$.

Part IV
References

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